FLUIDS

QUICK REFERENCE: Important Terms

absolute pressure the total static pressure at a certain depth in a fluid, including the pressure at the surface of the fluid
Archimedes principle the buoyant force acting on an object in a fluid is equal to the weight of the fluid displaced by the object
Bernoulli's principle the sum of the pressures exerted by a fluid in a closed system is constant
density the ratio of the mass to the volume of a substance
flow rate continuity the volume or mass entering any point must also exit that point
fluid any substance that flows, typically a liquid or a gas
gauge pressure the difference between the static pressure at a certain depth in a fluid and the pressure at the surface of the fluid
hydrodynamics the study of fluids in motion
hydrostatics the study of fluids at rest
ideal fluid a noncompressible, nonviscous fluid which exhibits steady flow, that is, the velocity of the fluid particles is constant
liquid substance which has a fixed volume, but retains the shape of its container
pressure force per unit area: the SI unit for pressure equal to one newton of force per square meter of area

Equations and Symbols

$$P = \frac{F}{A}$$

$$\rho = \frac{m}{V}$$

$$P_{depth} = P_0 + \rho g h$$

$$F_B = W_{fluid} = \rho g V_{fluid}$$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ (mass flow rate)

$A_1 v_1 = A_2 v_2$ (volume flow rate)

$$P_1 + \frac{1}{2}\rho v_1^{\,2} + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^{\,2} + \rho g y_2$$

where

P = pressure
F = force perpendicular to a surface
A = area
ρ = density
m = mass
V = volume
F_B = buoyant force
W = weight
g = acceleration due to gravity
v = speed or velocity
y = height above some reference level

Mass Density, and Pressure

The mass density ρ of a substance is the mass of the substance divided by the volume it occupies:

$$\rho = \frac{m}{V}$$

A *fluid* is any substance that flows and conforms to the boundaries of its container. A fluid could be a gas or a liquid; however on the AP Physics B exam fluids are typically liquids which are constant in density. An *ideal fluid* is assumed

- to be incompressible (so that its density does not change),
- to flow at a steady rate,
- to be non-viscous (no friction between the fluid and the container through which it is flowing), and
- flows irrotationally (no swirls or eddies).

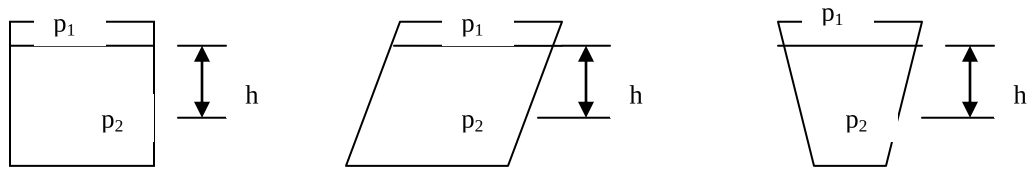
Any fluid can exert a force perpendicular to its surface on the walls of its container. The force is described in terms of the pressure it exerts, or force per unit area:

$$p = \frac{F}{A}$$

Pressure and Depth in a Static Fluid

The SI unit for pressure is the *Newton per meter squared*, or the *Pascal*. Sometimes pressure is measured in *atmospheres* (atm). One atmosphere is the pressure exerted on us every day by the earth's atmosphere. The relationship between one atmosphere and Pascals is
$1\ atm = 1.013 \times 10^5\ Pa$

A static (non-moving) fluid produces a pressure within itself due to its own weight. This pressure increases with depth below the surface of the fluid. Consider the containers of water with the surface exposed to the earth's atmosphere.



The pressure p_1 on the surface of the water is 1 atm, or 1.0×10^5 Pa. If we go down to a depth h below the surface, the pressure becomes greater by the product of the density of the water ρ, the acceleration due to gravity g, and the depth h. Thus the pressure p_2 at this depth is

$$p_2 = p_1 + \rho g h$$

In this case, p_2 is called the *absolute* pressure. The difference in pressure between the surface and the depth h is

$$p_2 - p_1 = \rho g h$$

This difference in pressure is called the *gauge* pressure. Note that the pressure at any depth does not depend of the shape of the container, only the pressure at some reference level (like the surface) and the vertical distance below that level.

Archimedes Principle

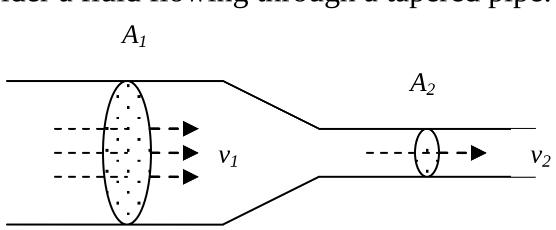
Archimedes principle allows us to calculate the *buoyant force* acting on an object in a fluid. The buoyant force is the upward force exerted by the fluid on the object in the fluid, and is equal to the weight of the fluid which is displaced by the object. For example, if a floating object displaces one liter of water, the buoyant force acting on the object is equal to the weight of one liter of water, which is about 10 N.
The buoyant force acting on an object in a fluid can be found by the equation

$$F_{buoyant} = \rho g V_{fluid\,displaced}$$

where ρ is the density of the fluid, g is the acceleration due to gravity, and V is the volume of the displaced fluid. If the buoyant force acting on an object in a fluid is equal to the weight of the object, the object will float.

The Equation of Continuity

Consider a fluid flowing through a tapered pipe:



The area of the pipe on the left side is A_1, and the speed of the fluid passing through A_1 is v_1. As the pipe tapers to a smaller area A_2, the speed changes to v_2. Since mass must be conserved, the mass of the fluid passing through A_1 must be the same as the mass of the fluid passing through A_2. If the density of the fluid is ρ_1, and the density of the fluid at A_2 is ρ_2, the *mass flow rate* through A_1 is $\rho_1 A_1 v_1$, and the mass flow rate through A_2 is $\rho_2 A_2 v_2$. Thus, by conservation of mass,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This relationship is called the *equation of continuity*. If the density of the fluid is the same at all points in the pipe, the equation becomes

$$A_1 v_1 = A_2 v_2$$

The product of area and the velocity of the fluid through the area is called the *volume flow rate*.

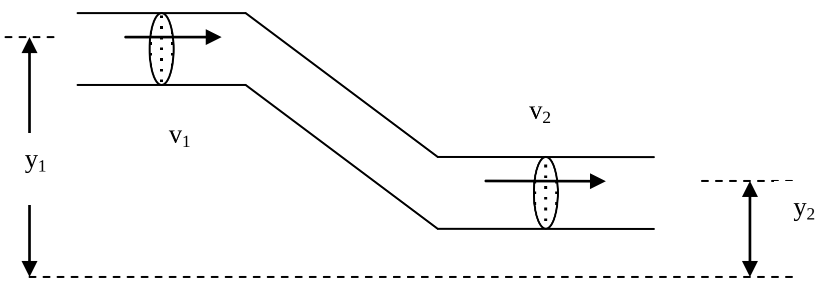
Bernoulli's Equation and Applications of Bernoulli's Equation

Recall that in the absence of friction, the total mechanical energy of a system remains constant, that is,

$$U_1 + K_1 = U_2 + K_2$$

$$mgy_1 + ½\, mv_1^2 = mgy_2 + ½\, mv_2^2$$

Bernoulli's principle states that the total pressure of a fluid along any tube of flow remains constant. Consider a tube in which one end is at a height y_1 and the other end is at a height y_2:



Let the pressure at y_1 be p_1 and the speed of the fluid be v_1. Similarly, let the pressure at y_2 be p_2 and the speed of the fluid be v_2. If the density of the fluid is ρ, Bernoulli's equation is

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

This equation states that the sum of the pressure at the surface of the tube, the dynamic pressure caused by the flow of the fluid, and the static pressure of the fluid due to its height above a reference level remains constant. Note that if we multiply Bernoulli's equation by volume, it becomes a statement of conservation of energy.

If a fluid moves through a horizontal pipe ($y_1 = y_2$), the equation becomes

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

This equation implies that the higher the pressure at a point in a fluid, the slower the speed, and vice-versa. The equation of continuity and Bernoulli's principle are often used together to solve for the pressure and speed of a fluid, as the following review questions illustrate.