Electrostatics: Not moving. Deals a lot with point charges, small ***q***. Can calculate the force between two point charges and the strength of the electric field created by point charges. If there are a number of charges you simply do the calculations for each charge separately and then add the results, the principle of superposition. However, never plug in minus charges for ***q***. Always plug in the positive magnitude for the charge whether positive or negative. Use the signs on the charges to tell you if they are moving right (positive) or left (negative). Assign each completed calculation of ***F*** or ***E*** the appropriate sign and then add the ***F's*** for the total force or add the ***E's*** for the total electric field strength.

$$F_E = k\frac{q_1 q_2}{r^2} \qquad \left(F = \sum k \frac{q_1 q_2}{r^2} \right) \qquad E = \frac{F_E}{q} \qquad F_E = Eq \qquad E = k\frac{q}{r^2} \qquad \left(E = \sum k \frac{q}{r^2} \right)$$

Remember small ***q*** is for a point charge, like a small group of electrons or protons, or even an oil drop.
Large ***Q*** (similar to heat energy, so watch out) is for lots and lots of charge, like that stored on charged plates.

Gravitational Fields	*Electric Fields*
$\textasteriskcentered\, F_g = G\frac{m_1 m_2}{r^2}$	$\textasteriskcentered\, F_E = k\frac{q_1 q_2}{r^2}$
$F_g = mg$	$\textasteriskcentered\, E = \frac{F_E}{q} \qquad F_E = qE$
Combine the above equation to create an new equation	**Combine the above equation to create an new equation**
$mg = G\frac{m_1 m_2}{r^2}$	$qE = k\frac{q_1 q_2}{r^2}$
Simplify	**Simplify**
$g = G\frac{m}{r^2}$	$E = k\frac{q}{r^2}$
F_g, Force of gravity is felt in a gravitational field. ***m***, mass causes gravitational fields ***g*** is a way to assess the strength of the gravity field. - Double mass: ***F_g*** (force) and ***g*** (field strength) double. - Distance follows the inverse square law. Double ***r***. ***F_g*** (force) and ***g*** (field strength) are 1/4. Halve ***r***. ***F_g*** (force) and ***g*** (field strength) are quadrupled.	***F_E***, Force of electrostatics, is felt in an electric field. ***q***, charge, causes electric fields. ***E*** is a way to assess the strength of the electric field. - Double charge: ***F_E*** (force) and ***E*** (field strength) double. - Distance follows the inverse square law. Double ***r***. ***F_E*** (force) and ***E*** (field strength) are 1/4. Halve ***r***. ***F_E*** (force) and ***E*** (field strength) are quadrupled.

Superposition
What is (a) the force of gravity and (b) the acceleration of gravity on a 1000 *kg* space ship located half way between the earth and the moon?
 	
E ◯   ●   ◯ M
 	
Both are pulling on it so find each ***F*** and ***g*** separately.
 	
(a) $F_g = G\frac{m_1 m_2}{r^2}$ $F_g\,earth = 6.67\times10^{-11}\frac{(5.98\times10^{24})(1000)}{\left[1/2(3.84\times10^8)\right]^2} = 10.2N$
 	
$F_g\,moon = 6.67\times10^{-11}\frac{(7.4\times10^{22})(1000)}{\left[1/2(3.84\times10^8)\right]^2} = 0.13N$
 	
As drawn: earth pulls left, negative
    moon pulls right, positive.
Add $-10.2N + 0.13N = -10.07N$ **left, so toward earth.**
 	
(b) $g = G\frac{m}{r^2}$ $g_{earth} = 6.67\times10^{-11}\frac{(5.98\times10^{24})}{\left[1/2(3.84\times10^8)\right]^2} = 0.0102\,m/s^2$
 	
$g_{moon} = 6.67\times10^{-11}\frac{(7.4\times10^{22})}{\left[1/2(3.84\times10^8)\right]^2} = 0.00013\,m/s^2$
 	
As drawn: earth pulls left, negative
    moon pulls right, positive.
Add $-0.0102 + 0.00013 = -0.01007\,m/s^2$ **left, so toward earth.**
	Superposition
	What is (a) the force of electricity on a +1 *C* charge located half way between a –3 *C* charge and a +2 *C* charge separated by 2 *m*? (b) What is the electric field strength at this location due to the – 3 and +2 C charges only?
	
	-3 ◯ ◯ ◯ +2
	
	Both are pulling on it so find each ***F*** and ***g*** separately.
	
	(a) $F_E = k\frac{q_1 q_2}{r^2}$ $F_E = 9\times10^9\frac{(3C)(1C)}{\left[1/2(2m)\right]^2} = 27\times10^9\,N$
	
	$F_E = 9\times10^9\frac{(2C)(1C)}{\left[1/2(2m)\right]^2} = 18\times10^9\,N$
	
	As drawn: –3 charge attracts +1 charge to left, negative.
	    +2 charge repels +1 charge to left, negative.
	Add $-27\times10^9\,N + -18\times10^9\,N = -45\times10^9\,N$ **left toward –3.**
	
	(b) $E = k\frac{q}{r^2}$ $E_{from-3} = 9\times10^9\frac{(3C)}{\left[1/2(2m)\right]^2} = 27\times10^9\,N/C$
	
	$E_{from+2} = 9\times10^9\frac{(2C)}{\left[1/2(2m)\right]^2} = 18\times10^9\,N/C$
	
	As drawn: –3 attracts positive test charge left, negative.
	    +2 repels positive test charge left, negative.
	Add $-27\times10^9 + -18\times10^9 = -45\times10^9\,N/C$ **left toward –3.**

$U_g = mgh_B$ ● **B** ↑ + $U_g = mgh_A$ ● **A** ↓ − **——— Ground h= 0**	+ + + + + + + + + + ＋ **B** ↑ + $U_E = ?$ ↑ ＋ **A** ↓ − - - - - - - - - - - - **Ground d = 0**

	Electric Field is based on a positive test charge
- Gravitational Field acts downward. ***Gravity Field forces masses to move.*** - Point **A** has less potential energy than point **B**. - To lift it from **A** to **B** you must increase energy, add energy. You must do **+ W** - Work energy theorem says that **W = Δ Energy**. - Moving from **A** to **B** increases **U**. - The increase (change) in **U** equals the work. **U** = 0 at the ground, the lowest point. Release the mass at **B**. Mass accelerates toward ground. The velocity increases so, **K** increases while **U** decreases. The mass can do work on the way down.	- Electric Field acts away from positive & toward negative. ***Electric Field forces charges to move.*** - Point **A** has less potential energy than point **B**. - To move it from **A** to **B** you must increase energy, add energy. You must do **+ W** - Work energy theorem says that **W = Δ Energy**. - Moving from **A** to **B** increases **U**. - The increase (change) in **U** equals the work. **U** = 0 at the ground, the lowest point. Release the charge at **B**. Charge accelerates toward negative, away from positive. The velocity increases so, **K** increases while **U** decreases. The charge can do work along the way.

Mass falls down, and needs work to go up.	***But, Electricity has positives and negatives.***
	Negative charges are the opposite.
	   Work is needed to move them toward ground.
	   They have higher **U_E** closer to the negative plate.
	   They do work on their way to the positive plate.

Similarities					
	causes gravity	m	q	causes electricity	
	measure of field strength	g	E	measure of field strength	
	distance above ground	h	d	distance above ground	

Gravitational Potential Energy $U_g = mgh$	***Electric Potential Energy*** $U_E = qEd$
Gravity field is very weak and its change is irrelevant. <center>$\Delta U_g = mg\Delta h$</center>	Electric field is very strong and changes with distance. <center>$\Delta U_E = q\Delta(Ed)$</center>
Work Energy Theorem $W = \Delta Energy$ <center>$W = mg\Delta h$</center>	**Work Energy Theorem** $W = \Delta Energy$ <center>$W = q\Delta(Ed)$</center>

No equivalent	<center>$U_E = q \times \Delta(Ed)$</center>
<center>$U_g = m \times g\Delta h$ $? = g\Delta h$</center>	***Electric Potential*** $V = \Delta(Ed)$ ***** $E = \dfrac{V}{d}$ ***** $U_E = qV$

	From above	$U_E = q \times Ed$	$E = k\dfrac{q}{r^2}$
	From above	$E = k\dfrac{q}{r^2}$	$E = \dfrac{V}{d}$
	Combine	$U_E = q\left(k\dfrac{q}{r^2}\right)d$	$\dfrac{V}{d} = k\dfrac{q}{r^2}$
	Simplify	***** $U_E = k\dfrac{q_1 q_2}{r}$	$V = k\dfrac{q}{r}$ ***** $V = k\sum_i \dfrac{q_i}{r_i}$

Electrostatic Equation Summary

$$F_E = k\frac{q_1 q_2}{r^2} \qquad E = \frac{F_E}{q} \qquad U_E = qV = k\frac{q_1 q_2}{r} \qquad E = -\frac{V}{d} \qquad V = k\sum_i \frac{q_i}{r_i} \qquad E = k\frac{q}{r^2} \text{ (not given)}$$

Remember superposition can be used with a number of these even though only one is shown with a sum symbol.

Voltage: It is a way to express the difference (change) between potential enervgies of two charged plates or two parts of an electrical circuit.

Confusing Terms

Electric Field		***E***
Electric Potential Energy	**Potential Energy**	***U***
Electric Potential	**Potential Difference** **Potential**	***V***

The variable Voltage ***V***, measured in the units *V*, for Volts

What's the pattern? If it contains the word energy it is ***U***. If it contains the word potential without the word energy it is ***V***. Also watch out for the big ***V*** for voltage and the small ***v*** for velocity. Sometimes you use them in the same problem.

Electric Field Lines

An <u>imaginary</u> way to view the electric field, similar to the elevation lines on a map. On maps, steep slopes are indicated by lines close together. In a similar fashion a concentration of electric field lines indicates higher field strength. The lines are based on the direction a positive test charge will go, so field lines leave positive charges and enter negative charges. Field lines leave and enter surfaces perpendicular to the surface.
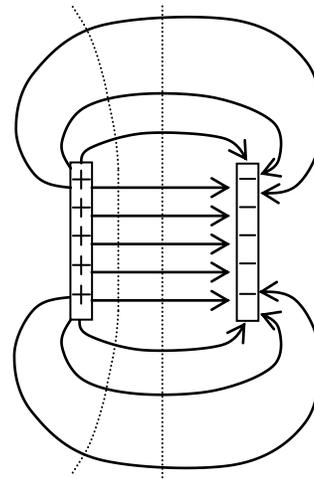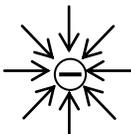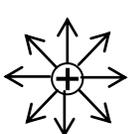
On the figure at the right the electric field lines are closest between the two plates. They are also virtually parallel in the center of the diagram.

Equipotential Lines: Lines of equal electric potential energy, ***U***. If a charge is moved along an equipotential line it does not change its potential energy. If you hike in the mountains along the contour lines you always stay at the same height above sea level, so you don't change your gravitational potential energy. The equipotential lines are perpendicular to the field lines. The two dotted lines in the diagram are two examples of the many possible equipotential lines.

Faraday Cage

Any enclosed metal structure, even one made of chicken wire, acts as a Faraday Cage. Charges pile up on the outer surface of the metal enclosure. Due to a combination of ***q*** and ***r*** at any point within the box, cylinder, sphere, etc. the electric field is zero inside. ***E*** = 0. This is why you are not electrocuted in a car or airplane if it is struck by lightening.

Capacitors or charged plates: Use large ***Q***.

Capacitors are a way to store electrical charge. They will hold an electrical charge as long as a voltage (potential difference) is applied to create the static electric build up on the plates, or until the plates are discharged by providing a path for the electrons to flow from one plate to the other.

Think of ***Capacitance***, ***C***, as the capacity of the capacitor to hold the charge. So it is the amount of charge divided by the potential difference that creates the charge build up. Since the charges would prefer to stay on the ground, ***U*** = 0, and they have been moved against the electric field to a higher ***U***, work must have been done by some kind of electrical pump to put them there. If work was done and they'd like to fall back to the ground (negative plate) then they have potential energy, ***U_c***, (C for capacitor).

$$C = \frac{Q}{V} \qquad\qquad U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

You can think of electricity as water. If you pump water up to a water tower it has high potential energy. If you turn off the pump it will flow back to the ground. A capacitor is an electrical water tower where charge is stored temporarily until it is needed. So if the electrical circuit needs to flush all its toilets simultaneously the capacitor discharges. It is refilled for the next big flush. ***Q*** is then the amount of water pumped into the tower. ***V*** is the potential difference between the height of the tower and ground level.

Why is $U_E = qV$ while $U_c = \frac{1}{2}QV$? As you move more electrons from one plate to the other the capacitor it becomes more charged. The first electron moves easily. But the second is repelled by the first, and the third by the first two. It becomes harder and harder to put more and more electrons on the plate since repulsion becomes higher and higher. Also the positive plate becomes more electron challenged, and pulls the electrons overloading the negative plate back to it. So the amount of charge moved starts out high and declines to zero. Average the highest with zero and you get half. If you pump water up to a water tower it goes fast at first. But, the weight of the water filling the tower begins to make it harder to pump, until the force of the water in the tower pushing down equal the force of the pump pushing up. Average the fast pumping at the start with zero pumping at the end, and you get half.

Point charges are not affected, since they are autonomous and not physically attached to another plate.

Current: The flow of electricity. Current, *I*, is assumed to be positive due to an old convention (Thanks a heap Ben!). We now know that the electrons actually flow, but we still refer to current as positive. So if we want to talk about actual electron flow we must say electron current or negative current, etc. The word current by itself implies positive current flow. Positive current flow follows the direction of the electric field, so this negative flow is counter to the electric field.

$I = \dfrac{\Delta Q}{\Delta t}$ It's a rate (divided by time). But, it is unlike velocity where we measure the distance the car went. Instead we stand still and count how many charges, *Q* (charge represents an amount of charged particles), go by. Big *Q* since we're counting lots-o-charge.

Resistance: When current flows the wires have internal properties that slow the current. When water flows down a stream it runs into rocks, sand, salmon on their way to their grave, etc. These act like friction slowing the water down. Resistance is like friction countering the forward progress of the electrons. Conductors slow the current very little, while insulators have a lot of resistance and slow it drastically. In addition to the wires, all appliances, and even the sources of electricity have resistance. There are also actual resistors built into circuits to help control the amount of electrical flow to exact quantities in various parts of the circuit.

$R = \dfrac{\rho\ell}{Area}$ Resistance is a function of resistivity, ρ, wire length, ℓ, and cross sectional area.

   Resistivity, ρ, is like the coefficient of friction. It is given since it is derived by experimentation. Different materials have different natural resistances. Gold has very low resistance, copper is pretty good, and plastic is not so hot.

So let's worry about length and cross section of wire.
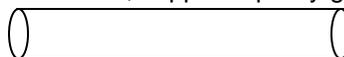
What kind of water pipe would provide the least friction, allowing water to pass through more easily?
a. long and narrow   b. long and wide   c. short and narrow   d. short and wide

Look at the equation above. Try to make *R* as small as possible, by changing ℓ and Area. Make length small and area big, so the answer is d. The longer the wire the more resistance it has. And resistance is like friction. What type of energy does some of the KE turn into when an object is slowed by friction? Heat. What kind of energy is produced when charges are slowed down by resistance? Heat. What do you feel when you touch an electrical component, like a stereo? It gets hot. Heat loss is disadvantageous. You're losing valuable energy, wasting money on you electrical bill, and increasing entropy. So minimizing resistance is usually a good thing. But, sometimes you need to create resistance if you have components that can only handle certain amounts of power, energy, voltage, etc. In addition the wires themselves act like resistors. One goal in circuit design is to shorten the wires between components to minimize power loss and heat. How much energy is lost to heat in the high tension wires from Hoover Dam to California?

Two important equations for circuits, and two equations (not given) but that can be derived from these.

$V = IR$ $P = IV$ $P = I^2R$ $P = \dfrac{V^2}{R}$

DC Circuits: Direct Current, meaning it travels in one direction only following the electric field lines. The battery or power supply pumps charges creating a potential difference (voltage) between the ends of the circuit. You have a positive terminal (positive plate) which is a region of high potential energy. At the other end of the circuit (wires and components) is a negative terminal (negative plate) which is a region of low potential energy. Positive current (positive charges) want to fall toward the ground through the potential difference toward the negative plate. This is the direction of the electric field. So the potential difference between the positive and negative plates creates the electric field that guides the charges through the circuit. (Just remember the electrons really flow, so its all backwards. But, mathematically you get the same numbers).

Water Analogy: A pump (battery or generator) pumps water (charge) from the ground (negative plate) up to the top of the water theme park (positive plate) The water (charge) wants to follow the gravitational field (electric field) back to the ground. But we've constructed a series of tubes (wires) for the water (charge) to pass through on the way to the ground. Because the water (charge) is losing potential energy as it falls, and because energy must be conserved, the potential energy must be turning into another form of energy. It is turning into the kinetic energy of the water (charge). If the water (charge) is run through a paddle wheel (appliance, light bulb, etc.) it can turn the wheel thus doing useful work (browning toast, lighting up you desk lamp). Unfortunately, some of the water is slowed going through the wheel (appliance, lamp, etc.) and this friction (resistance) creates heat. Also our contractor who built the park used inferior materials that impede (high resistivity) the flow of water (current). So the water (charge) is slowed in the tubes (wires) as well. Fortunately the contractor did note that the paddle wheel (appliance, lamp, etc.) was very small and fragile. And while the plan placed the paddle wheel (appliance, lamp, etc.) in a very steep section of pipe (high potential difference) where the flow of water (current) was very high the contractor installed a metal grating (resistor) in front of the paddle wheel (appliance, lamp, etc.) to slow the amount of water flow (current) to a level that the paddle wheel (appliance, lamp, etc.) could handle.

Circuits Containing Resistors

Series: All resistors are in line. Resistors are like sections of wire, designed to slow current. Let's examine the resistivity of two sections of wire placed in series, or end to end.

$$R = \frac{\rho\ell}{Area}$$

ℓ is doubled, but cross sectional area stayed the same. So **R** doubles. If there were three wires **R** would triple. This makes sense. Lets pretend they make your commute on the freeway longer, but they didn't build more lanes to widen the road. So you experience the same traffic (resistance) for two or three times as long.

So in series Resistance adds $\qquad R_s = \sum_i R_i \qquad$ or $\qquad R_s = R_1 + R_2 + R_3 + ...$

Now let's examine the resistivity of two sections of wire placed in parallel, or next to each other.

$$R = \frac{\rho\ell}{Area}$$

ℓ stays the same, but cross sectional area doubles. It's more complicated than above. Area is in the denominator. If area doubles **R** is cut in half. If there were three wires **R** would be cut in third. This makes sense since the current and a bigger pipe to flow through. Its like they expanded the freeway from four lanes to 8 or even 12. You travel the same distance as before but now have less traffic congestion.

So in parallel $\qquad \frac{1}{R_p} = \sum_i \frac{1}{R_i} \qquad$ or $\qquad \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$

Please don't solve for 1/R. Remember to invert you final answer.

Current and Voltage in Circuits: Think of current as traffic and resistors as less lanes

Series   Current stays the same (resistors in line slow traffic in the whole circuit) $I_s = I_1 = I_2 = I_3 = ...$

           Voltage adds $V_s = V_1 + V_2 + V_3 + ...$ (cars are all pushed down a single path)

Parallel   Current adds $I_s = I_1 + I_2 + I_3 + ...$ (current can choose paths, but the total must split between paths available)

           Voltage stays the same $V_s = V_1 = V_2 = V_3 = ...$ (cars have an equal pressure down any path)

Circuits containing Capacitors

The rules for capacitors are **_opposite_** the rules for resistors. But similar problem solving techniques apply.

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i} \qquad\qquad\qquad C_P = \sum_i C_i$$

Magnetism

Similar to electricity, in that unlike poles attract. Different in that, separate poles cannot exist by themselves. Magnetism and Electricity are interconnected. Electric fields can influence magnets, and magnetic fields can influence moving charged particles and current carrying wires.

B is a measure of magnetic field strength, analogous to **E** for electric fields, and **g** for gravitational fields.

$F_B = qvB\sin\theta$ charged particles (small **q**, point charge) traveling (velocity, **v**) in a magnetic field (**B**)

$F_B = BI\ell\sin\theta$ influence of a magnetic field (B) on a current (I) carrying wire of a certain length (ℓ)

Right Hand Rule   Fingers (plural) represent the magnetic field lines (plural).
           Thumb represents the charged particles velocity, or the direction of current flow in a wire. You want to hitchhike with the charged particle or current flow in the wire.
           Palm, represents the direction of force. A slap from the palm of the hand is a force.

All are perpendicular to each other. The force on a charged particle in a magnetic field is always perpendicular, so _no work is done_ in this case. They ask this on just about every AP test somewhere.

A charged particle flying in a magnetic field will move in a _circular path_.

What is causing the circular motion? F_B. So $\qquad F_C = F_B \qquad\qquad m\frac{v^2}{r} = qvB$

If magnetism can causing a current carrying wire to move, then could moving a current carrying wire in a magnetic field cause current to flow?

Electromagnetic Induction:
A loop of wire moving in a magnetic field experiences a change in flux.

$\phi = B \cdot Area$ **Flux:**   The strength of a magnetic field moving through an area of space, such as a loop of wire.

To generate electricity you **_need a changing flux_**. Take a loop of wire and move it through or rotate it in a magnetic field. The area is the area of the loop of wire.

$\varepsilon_{avg} = -\dfrac{\Delta \phi_m}{\Delta t}$ **Induction:**   The changing flux, means the charged electrons in the wire are experiencing a changing magnetic field. This causes them to be deflected, and since they are bound by the wire in the loop they travel down the wire. The **minus sign** in this equation is a **reminder that the moving electrons create their own magnetic field** (Moving charge is the basis of electromagnetic fields), and that **this field is opposite in direction to the magnetic field causing the flux**. This is **Lenz's Law**. It means you use the right hand rule as before, but you reverse the direction of current flow in you final answer. Remember you only do this reversal in electromagnetic induction.

If you have a bar moving along two wires through a magnetic field you can use the following equation

$\varepsilon = B\ell v$ ℓ is the length of the bar, and v is the velocity of the bar.

This is derived using the formulas above.

$$\varepsilon_{avg} = -\dfrac{\Delta \phi_m}{\Delta t} \qquad \phi = B \cdot Area \qquad \varepsilon_{avg} = \dfrac{B \cdot Area}{\Delta t} \qquad \varepsilon_{avg} = \dfrac{B \cdot \ell \times d}{\Delta t} \qquad \varepsilon_{avg} = B \cdot \ell \dfrac{d}{\Delta t} \qquad \varepsilon = B\ell v$$

Electromotive Force, emf:
Induction actually creates electromotive force ε, which really isn't a force.
Every device that generates electricity has resistance, known as **internal resistance**. So in a way **emf** is the amount of voltage you could produce if there were no internal resistance in a battery, power supply, generator, or wire loop creating electricity.

So the equation for voltage induced is actually $V = emf - internal\ resistance$.

But, in the problems you are given internal resistance will usually be negligible, so voltage and **emf** are essentially the same. $V = emf - 0$ In AP B this is normally the case. But, not in college.

If it is not zero, just use you're rules for series and parallel and treat the internal resistance as an ordinary resistor that happens to be right next to the battery or power supply.

Watch these.
Q   vs. **q**
E   in electricity is electric field, later it is energy.
U_E   is the electric potential energy or potential energy
V   is the potential difference, electric potential, potential, or voltage
V   vs. **v**

The magnetic field does no work on charged particles since everything is perpendicular.

Work, however is very useful

$$W = \Delta U_E = qV \qquad\qquad W = \Delta U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$P = IV = \frac{W}{t} \qquad\qquad P = I^2 R = \frac{W}{t} \qquad\qquad P = \frac{V^2}{R} = \frac{W}{t}$$

And a whole bunch of combined equations that I can't even begin to list.

Conservation of energy
They love questions with charged particles moving between charged plates.

$$U_{Ef} + K_f = U_{Ef} + K_f \qquad\qquad qV + \frac{1}{2}mv^2 = qV + \frac{1}{2}mv^2$$

$qV = \dfrac{1}{2}mv^2$ If it is not moving at the beginning and then uses up all the **PE** converting it entirely into **KE**