8.6: 2-D Collisions

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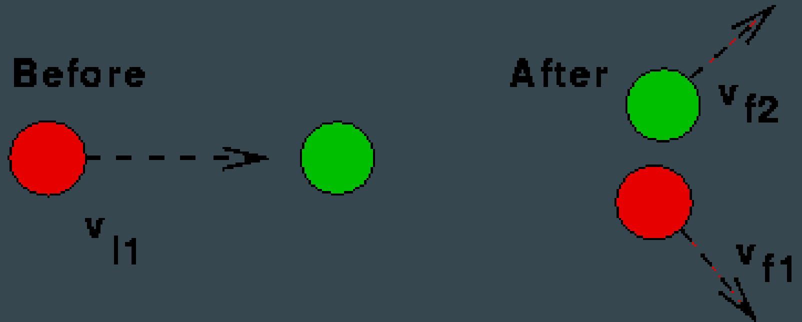
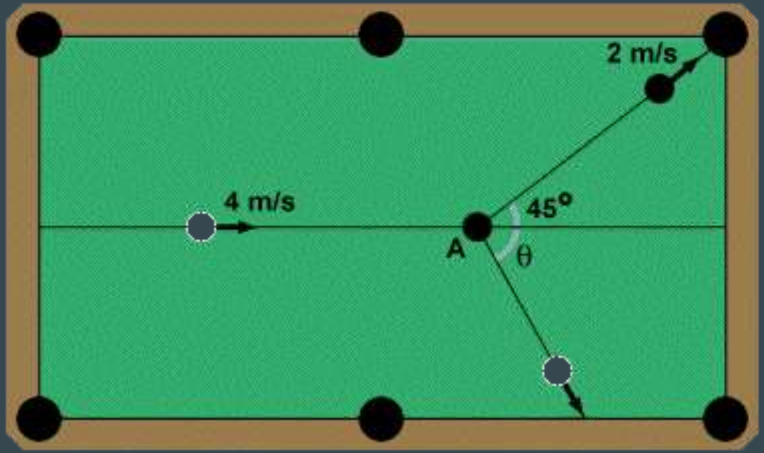


What is a 2-D collision?

A 2-D collision occurs when one moving object collides with a second object causing both to scatter in new directions. Both of these objects are now traveling at an angle with a velocity.

A real world application of this is two billiard ("pool") balls hitting each other.



Two dimensional collision in coordinate system

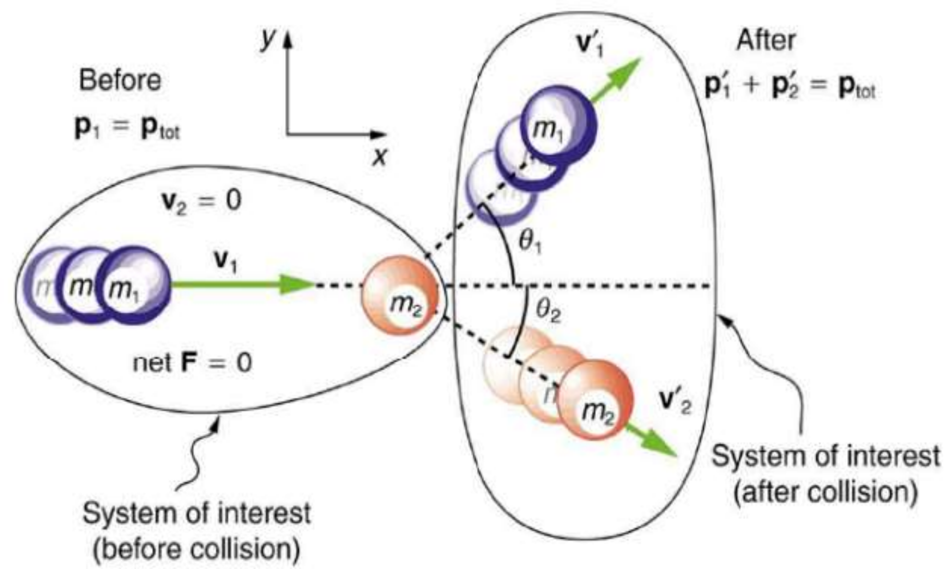
Assuming zero net external force, and the total momentum is a conserved quantity. Unlike 1-D collisions, we now treat the momentum as a *vector* quantity.

F_{net}= 0, so that momentum **p** is conserved.

- m_2 is initially at rest (second object)
- v_1 is parallel to the x axis

(This coordinate system sometimes

referred to as laboratory system)



Conservation of Momentum x-axis.

Before the collision, the total “x” momentum is simply m1vi1, since the second object is at rest, and the first object is moving along the x-axis with speed vi1. After the collision, the x-momentum of the first $m_1 v_{f1} \cos\theta_1$, which is m1 times the component of the first object's final velocity. Momentum of second $m_2 v_{f2} \cos\theta_2$

Hence,

Conservation of Momentum along the x-axis

$$m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$$

Conservation of momentum y-axis.

Before collision, y-momentum is zero. After collision, y-momentum is $-m_1 v_{f1} \sin\theta_1$ which is m1 times y-component final velocity. Y-momentum of second object is $m_2 v_{f2} \sin\theta_2$

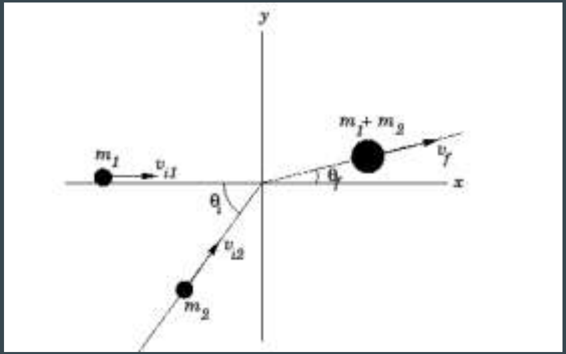
Hence

Conservation of Momentum along the y-axis

$$0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2$$

Elastic collision

For the special case of elastic collision, we can equate the total kinetic energies of two objects before and after the collision.



Hence,

$$\frac{1}{2} m_1 v_{i1}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2 .$$

Application Problem

47. A 3000-kg cannon is mounted so that it can recoil only in the horizontal direction. (a) Calculate its recoil velocity when it fires a 15.0-kg shell at 480 m/s at an angle of 20.0° above the horizontal. (b) What is the kinetic energy of the cannon? This energy is dissipated as heat transfer in shock absorbers that stop its recoil. (c) What happens to the vertical component of momentum that is imparted to the cannon when it is fired?

a)Magnitude of its recoil velocity when it fires a 15 kg shell at 480 m/s at an angle of 20°

mass of shell = 15 kg

initial horizontal speed of shell 480(cos20°) = 451m/s

initial horizontal momentum of shell = (15)(451) = 6766 kgm/s

horizontal recoil velocity of cannon = 6766/3000 = 2.26 m/s

b) What is the kinetic energy of the cannon? This energy is dissipated as heat in the shock absorbers that stop its recoil.

Initial KE of the cannon as the shell leaves the barrel =

1/2mV^2

= (.5)(3000)(2.26)^2 = 7661.4 J

c) What happens to the vertical component of momentum that is imparted to the cannon when it is fired?

The ground will exert a normal force to oppose recoil of the cannon in the vertical direction. The momentum in the vertical direction is transferred to the earth. The energy is transferred into the ground, making a dent where the cannon is. After long barrages, cannon will have unpredictable aim because the ground is full of divots.

Math Problem

Q: Two identical pucks collide on an air hockey table. One puck was originally at rest. If the incoming puck has a speed of 6.00 m/s and scatters to an angle of 30.0°, what is the velocity of the second puck?

Step 1: Understand the momentum after collision must equal the momentum before.

u = speed of the original moving puck after collision (moving at 30 degrees to X)

v = speed of the other puck after collision (moving at 60 degrees to X)

m = mass of each puck

$$m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$$

$$0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2$$

Air hockey problem cont.

X direction.

Using earlier conservation equation you get the before= M*6 and after=M*cos(30)*u+ m*cos(60)*v

Then equate before and after: m*6 = m*cos(30)*u + m*cos(60)*v

6 = u*sqrt(3)/2 + v/2

Y direction.

Using Y direction formula,

m*sin(30)*u = m*sin(60)*v

u/2 = v*sqrt(3)/2 ..... u = v*sqrt(3)

Combine the equations from before and after.

6 = v*sqrt(3)*sqrt(3)/2 + v/2

6 = v*3/2 + v/2 = 2*v

v = 3 and u = 3*sqrt(3)

LAB & Interactive Activity

Collision of two billiard balls:

https://www.youtube.com/watch?v=gVM7wGmhmV8

Example of Billiard ball collisions and the velocity vectors:

http://yteach.com/page.php/resources/view_all?id=vector_displacements_velocity_decmposition_forces_page_4&from=search

Reiteration of what to know.

8.6 Collisions of Point Masses in Two Dimensions

- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the x-axis parallel to the velocity of the incoming particle.
- Two-dimensional collisions of point masses where mass 2 is initially at rest conserve momentum along the initial direction of mass 1 (the x-axis), stated by $m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$ and along the direction perpendicular to the initial direction (the y-axis) stated by $0 = m_1 v'_{1y} + m_2 v'_{2y}$.
- The internal kinetic before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1^{\,2} = \frac{1}{2}mv'^{\,2}_1 + \frac{1}{2}mv'^{\,2}_2 + mv'_1 v'_2 \cos(\theta_1 - \theta_2).$$

- Point masses are structureless particles that cannot spin.

https://www.youtube.com/watch?v=esf81_K-uT8